

Possible Classification of the Chiral Scalar σ -Nonet

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We recently observed the iso-singlet scalar $\sigma(600)$ -particle and the iso-doublet scalar $\kappa(900)$ -particle, through the re-analyses of $\pi\pi$ - and $K\pi$ -scattering phase shifts, respectively. First, assuming the two iso-singlet states, the $\sigma(600)$ and the established $f_0(980)$, being the ideal mixing states $n\bar{n}$ and $s\bar{s}$, respectively, of a single scalar nonet, it is pointed out that the mass value of the iso-singlet flavor-octet state, obtained from the orthogonal transformation, satisfies, together with the $\kappa(900)$ and the iso-triplet scalar $a_0(980)$ -particle, the Gell-Mann Okubo mass-formula. Furthermore, it is argued that, by investigating their properties of masses and widths, this scalar σ -nonet, together with the pseudo-scalar π -nonet, realizes the linear representation of $SU(3)$ chiral symmetry.

§1. Introduction

—Observation of $\sigma(600)$ and $\kappa(900)$ —

In previous works by our group, we have observed the scalar $q\bar{q}$ -mesons with $I=0$ and $I=1/2$, $\sigma(600)^{1)-4)}$ and $\kappa(900)^{5),3),4)}$ respectively, by the re-analyses^{*)} of the $\pi\pi$ - and $K\pi$ -scattering phase shifts. The mass and width of the $\sigma(\kappa)$ -meson are determined with the values of $m_\sigma = 540 - 675$ MeV^{**) and $\Gamma_{\sigma \rightarrow \pi\pi} = 385 \pm 70$ MeV ($m_\kappa = 905^{+65}_{-30}$ MeV and $\Gamma_{\kappa \rightarrow K\pi} = 545^{+235}_{-110}$ MeV). Existence of these scalar mesons has been conventionally neglected¹¹⁾⁻¹³⁾ for many years. The reason which led us to different results using the same data^{14),15)} are two-fold: Technically, we have applied a new S -matrix parametrization method, the interfering Breit-Wigner amplitude (IA-method)¹⁾⁻⁵⁾ for the analyses, where the amplitude is represented directly by the physically meaningful parameters, masses and widths of resonances, while physically, we have introduced a negative background phase $\delta_{BG}^{1)-5),16)}$ of a hard core type.¹⁷⁾⁻¹⁹⁾ This δ_{BG} represents a very strong repulsive force between pions (or pion and kaon), which strongly cancels²⁰⁾ the attractive force due to intermediate σ - (or κ -) production. The obtained χ^2 -value is greatly improved compared with that in the conventional analysis without the σ (and κ) meson. The physical reason¹⁶⁾ for missing the existence of these new scalar particles in the conventional phase shift analyses is due to overlooking this cancellation mechanism, which is guaranteed by current algebra and PCAC.}

In addition to these new particles, there are the other established scalar mesons below 1 GeV: the iso-singlet $f_0(980)$ and the iso-triplet $a_0(980)$. The purpose of this paper is to investigate the possibility for classification of these scalar particles into a

^{*)} Other recent phase shift analyses also suggest the existence of light $\sigma^{6)-9)}$ and $\kappa^{10)}$ particles.

^{**) In a previous work²⁾ we estimated m_σ with the value of 585 ± 20 MeV for the “standard” phase shift of the CERN-Münich b-analysis.¹⁴⁾}

single scalar nonet, and furthermore, to study whether it is possible to ascribe this scalar σ -nonet to the chiral partner of the pseudo-scalar π -nonet in $SU(3)$ chiral symmetry.

In §2, the Gell-Mann Okubo (GMO) mass formula is shown to be approximately satisfied for the octet members of this scalar σ -nonet, where, in order to identify the octet members, the OZI-rule concerning the decay property of $f_0(980)$ is assumed. In §3.1, by investigating the relations among their masses and widths, they are suggested to have the properties of the scalar nonet predicted by the $SU(3)$ linear σ model ($L\sigma M^{21)-25}$). Being based on these results, in §3.2 the situations of the validity of OZI rule, which was assumed in §2, is examined somewhat quantitatively. Section 4 is devoted to concluding remarks.

§2. Scalar σ -nonet mass formula

Experimentally, $f_0(980)$ has a considerably small $\pi\pi$ -width regardless of its large phase volume, while having a rather large $K\bar{K}$ -width in spite of the fact that its mass is quite close to the $K\bar{K}$ -threshold. Assuming the approximate validity of the OZI rule, this fact seems to suggest that $f_0(980)$ consists of almost pure $s\bar{s}$ -component. Here we simply assume that $\sigma(600)$ and $f_0(980)$ are the ideal mixing²⁶⁾ states of a single scalar nonet and that the squared-mass matrix takes a diagonal form in these ideal bases. The ideal-mixing states are related to the octet state σ_8 and the singlet state σ_1 through the orthogonal transformation:

$$\begin{pmatrix} \sigma(600) \\ f_0(980) \end{pmatrix} = \begin{pmatrix} \sigma_n \\ \sigma_s \end{pmatrix} = O \begin{pmatrix} \sigma_8 \\ \sigma_1 \end{pmatrix}, \quad \sigma_n \equiv \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}, \quad \sigma_s \equiv s\bar{s}, \quad (2.1)$$

where O is the matrix of the orthogonal transformation, given by

$$O \equiv \begin{pmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix}. \quad (2.2)$$

Through the transformation O , the elements of the squared-mass matrix in the octet-singlet bases are numerically given by

$$\begin{aligned} \begin{pmatrix} m_{\sigma_8}^2 & m_{\sigma_{81}}^2 \\ m_{\sigma_{81}}^2 & m_{\sigma_1}^2 \end{pmatrix} &= {}^tO \begin{pmatrix} m_{\sigma(600)}^2 & 0 \\ 0 & m_{f_0(980)}^2 \end{pmatrix} O \\ &= \begin{pmatrix} (0.87 \text{ GeV})^2 & -(0.54 \text{ GeV})^2 \\ -(0.54 \text{ GeV})^2 & (0.74 \text{ GeV})^2 \end{pmatrix}, \quad (2.3) \end{aligned}$$

where we have used the experimental values $m_\sigma=0.59 \text{ GeV}$ and $m_{f_0(980)}=0.98 \text{ GeV}$.^{*)}

^{*)} The fact that m_{σ_1} is smaller than m_{σ_8} is in contrast with the case of pseudoscalar η - η' mass splitting. This fact reflects the property of the $U_A(1)$ -breaking interaction (see, §3). In this connection, note that the famous nonet mass formula²⁶⁾ of the vector mesons, $m_\rho^2 = m_\omega^2$ and $m_\phi^2 - m_{K^*}^2 = m_{K^*}^2 - m_\rho^2$, is valid in the case $m_{V_1}^2 = m_{V_8}^2$.

The mass m_{σ_8} can also be determined theoretically by using the Gell-Mann Okubo (GMO) relation,

$$m_\kappa^2 = (3m_{\sigma_8}^{\text{theor}^2} + m_{a_0}^2)/4, \quad (2.4)$$

as

$$m_{\sigma_8}^{\text{theor}} = 0.88 \text{ GeV}, \quad (2.5)$$

where we have used the experimental values $m_\kappa=0.91 \text{ GeV}$ and $m_{a_0(980)}=0.98 \text{ GeV}$. This value of $m_{\sigma_8}^{\text{theor}}$ is quite close to $m_{\sigma_8}=0.87 \text{ GeV}$, obtained phenomenologically in Eq. (2.3). This fact supports our classification that $\sigma(600)$, $\mathbf{f}_0(980)$, $\kappa(900)$ and $\mathbf{a}_0(980)$ form a single scalar nonet.

§3. Chiral symmetry and properties of the σ -nonet in relation with the π -nonet

3.1. Chiral symmetry and the mass and width of the scalar nonet

We assume that our scalar σ -nonet is a composite $q\bar{q}$ -system as a chiral partner of the pseudoscalar π -nonet and that in the low energy region, where the structure of composite mesons can be neglected, they can *effectively* be described by the linear σ model ($\text{L}\sigma\text{M}$). In the matrix notation $B \equiv s + i\phi$ ($s \equiv \lambda^i s^i/\sqrt{2}$ and $\phi \equiv \lambda^i \phi^i/\sqrt{2}$ denoting the scalar and pseudoscalar meson nonet, respectively), the Lagrangian of $SU(3)\text{L}\sigma\text{M}^{25), 24)}$ is

$$\begin{aligned} \mathcal{L}^{\text{L}\sigma\text{M}} = & \frac{1}{2} \langle \partial_\mu B \partial^\mu B^\dagger \rangle - \frac{\mu^2}{2} \langle BB^\dagger \rangle - \frac{\lambda_1}{4} \langle BB^\dagger \rangle^2 - \frac{\lambda_2}{2} \langle (BB^\dagger)^2 \rangle \\ & + \kappa_d (\det B + \det B^\dagger) + \langle f s \rangle, \end{aligned} \quad (3.1)$$

where $\langle \rangle$ represents the trace. Here f is proportional to the current-quark mass matrix in QCD and the form $f = \text{diag}\{f_n, f_n, f_s\}$ guarantees the PCAC. In the process of spontaneous chiral symmetry breaking, s acquires the vacuum expectation value $s_0 \equiv \Sigma = \text{diag}\{a, a, b\}$, and $s\phi\phi$ -couplings appear. The pseudoscalar decay constants f_π and f_K , and their ratio are represented by

$$f_\pi = \sqrt{2}a, \quad f_K = \frac{a+b}{\sqrt{2}}; \quad \frac{f_K}{f_\pi} = \frac{a+b}{2a}. \quad (3.2)$$

The six model parameters contained in Eq. (3.1) are determined by the masses of π, η, η', σ and κ , and the decay constant f_π , and thus we can predict the masses and widths of the scalar mesons. These are given in Table I. The predicted properties are very sensitive to the value of f_K/f_π ,²⁴⁾ as shown in Fig. 1.^{24), 27)} The deviation of the value of f_K/f_π from 1 represents the degree of $SU(3)$ breaking by s_0 , as can be seen from Eq. (3.2). We prefer the region in which this ratio satisfies $1.329 < f_K/f_\pi < 1.432$ (which is somewhat larger than the experimental value 1.22) indicated by the two vertical lines in the figure, where the value m_κ^{theor} reproduces the experimental value within its uncertainty. In this region, Γ_σ and Γ_κ are obtained with much larger

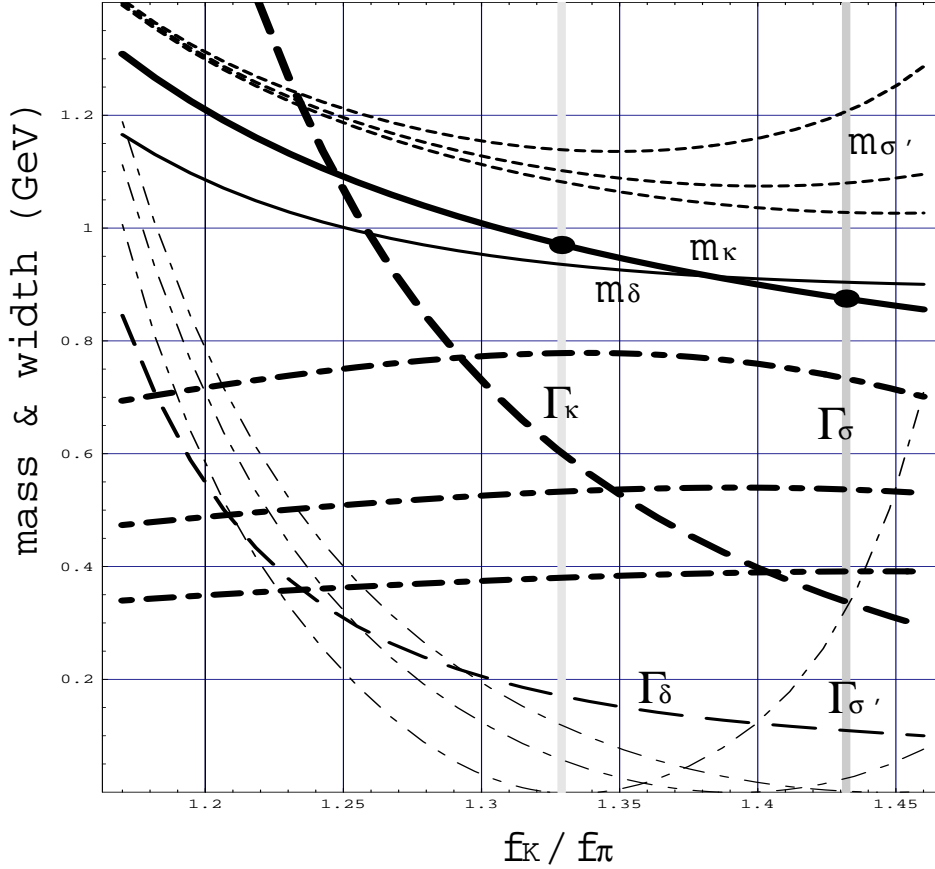


Fig. 1. The scalar meson masses and widths (GeV) versus f_K/f_π . The upper, middle and lower lines of $M_{\sigma'}$, Γ_σ and $\Gamma_{\sigma'}$ correspond, respectively, to the input values $M_\sigma = 650, 585$ and 535 MeV. This figure was extended from the original figure drawn by Chan and Haymaker,²⁴⁾ including the widths of scalar mesons.

values than those of $\Gamma_{\sigma'}$ and Γ_δ . The reason that $\Gamma_{\kappa \rightarrow K\pi}$, in spite of its comparatively smaller phase space, becomes as large as $\Gamma_{\sigma \rightarrow \pi\pi}$ is due to the contribution to the coupling constant $g_{\kappa K\pi}$ from the determinant-type interaction in Eq. (3.1).

The predicted widths of σ and κ are consistent with the experimental values. The predicted masses and widths of the other members, $\delta(I=1)$ and $\sigma'(s\bar{s})$, are close to those of $a_0(980)$ and $f_0(980)$, respectively. The σ' and δ states have large $K\bar{K}$ -coupling constants.^{*)} (Especially the σ' strongly couples to the $K\bar{K}$ channel.) This suggests that these states may also be interpreted as $K\bar{K}$ -molecule states.²⁹⁾

In L σ M, σ and δ have almost the same quark content. Despite this fact, m_δ

^{*)} In the case $f_\pi = 93$ MeV, $f_K/f_\pi = 1.394$ and $m_\sigma = 585$ MeV, for σ' , $\mathcal{L}_{\text{int}} = g_{\sigma'\pi\pi}\sigma'\pi^2 + g_{\sigma'KK}\sigma'(K^+K^- + K^0\bar{K}^0)$, $g_{\sigma'\pi\pi} = -0.02$ GeV, and $g_{\sigma'KK} = -4.97$ GeV. For δ , $\mathcal{L}_{\text{int}} = g_{\delta\pi\eta}(\pi^-\delta^+ + \pi^+\delta^- + \pi^0\delta^0)\eta + g_{\delta KK}(\delta^0\frac{K^+K^- - K^0\bar{K}^0}{\sqrt{2}}) + \delta^+K^0K^- + \delta^-\bar{K}^0K^+$, $g_{\delta\pi\eta} = -3.12$ GeV, and $g_{\delta KK} = -3.19$ GeV.

Table I. The properties of the scalar meson nonet predicted by $SU(3)L\sigma M$, compared with experiments. The underlined values of m_σ and m_κ along with f_π , m_π , m_η and $m_{\eta'}$ are used as inputs. The region of the value of m_κ^{exp} corresponds to the region in which the ratio of the decay constants satisfies $1.329 < f_K^{\text{L}\sigma M}/f_\pi^{\text{L}\sigma M} < 1.432$. The properties of δ and σ' become close to those of the observed resonances $a_0(980)$ and $f_0(980)$, respectively, taken as the experimental candidates. The quantity $\Gamma_{\sigma'}^{\text{theor}}$ is the partial width $\Gamma_{\sigma' \rightarrow \pi\pi}^{\text{theor}}$. The value of $\Gamma_{\sigma' \rightarrow K\bar{K}}^{\text{theor}}$ is highly dependent on $m_{\sigma'}^{\text{theor}}$, since $m_{\sigma'}^{\text{theor}}$ is close to $K\bar{K}$ -threshold.

	$m^{\text{theor}}/\text{MeV}$	$m^{\text{exp}}/\text{MeV}$	$\Gamma^{\text{theor}}/\text{MeV}$	$\Gamma^{\text{exp}}/\text{MeV}$
σ	<u>535 ~ 650</u>	<u>535 ~ 650</u>	400 ~ 800	385 ± 70
κ	905^{+65}_{-30}	905^{+65}_{-30}	300 ~ 600	545^{+235}_{-110}
$\delta = a_0(980)$	900 ~ 930	982.7 ± 2.0	110 ~ 170	57 ± 11
$\sigma' = f_0(980)$	1030 ~ 1200	993.2 ± 9.5	0 ~ 300	67.9 ± 9.4

becomes much larger than m_σ . This phenomenon is explained by the properties of the instanton-induced $U_A(1)$ -breaking determinant-type interaction.^{*)}

Thus, it may be plausible to regard **$\sigma(600)$, $\kappa(900)$, $a_0(980)$, and $f_0(980)$ as members of the scalar nonet, forming with the members of π -nonet a linear representation of the $SU(3)$ chiral symmetry.**^{**) (}

All the above results are obtained in the tree level bases of the $L\sigma M$. Following the renormalization procedure of Chan and Haymaker,²⁴⁾ we have made a preliminary estimate of the one-loop effects with the present renewed experimental data. The width at the one-loop level is defined from the imaginary part of the inverse propagator of the relevant particle, and it is almost equal to the value given in Table I. The masses of the scalar nonet (especially the $m_{\sigma'}$) is affected by the comparatively large one-loop effect, and $m_{\sigma'}^{1\text{-loop}}$ becomes much larger than $m_{f_0(980)}$. We might expect that introduction of the form factor, reflecting the composite structure of the relevant meson system, decreases the effects and leads to the improvement.

Another problem of our scalar assignment is, as mentioned above, our theoretical value of f_K/f_π is^{***)} somewhat larger than the experimental one.

3.2. The GMO mass formula, OZI rule and effective linear σ model

In §2 it is shown that the GMO mass formula is approximately satisfied for the octet members of our σ -nonet of scalar mesons, where $\sigma(600)$ and $f_0(980)$ are assumed to be ideal mixing states of isoscalars f_8 and f_1 . This assumption was motivated by reasoning concerning the decay properties of $f_0(980)$, based on the approximate validity of the OZI rule.

In §3.1, by using the $L\sigma M$ at the tree level, the properties of the σ -nonet are analyzed quantitatively. In this sub-section, based on the results of §3.1, the validity of the GMO mass formula and the OZI rule in $L\sigma M$ is examined.

In the $L\sigma M$ Eq. (3.1), since the explicit symmetry breaking term $\langle fs \rangle$ is introduced in the T_3^3 -breaking pattern, the GMO mass formula is expected to be

^{*)} However, see another viewpoint on this problem given by Jaffe.³⁰⁾

^{**) This interesting assignment was suggested and insisted upon repeatedly by M. D. Scadron.²⁸⁾}

^{***) A possible solution may be given by taking into account the pseudoscalar-axial vector mixing effect, which induces the field renormalization of pseudoscalars (see Ref. 16)).}

satisfied.^{*)} On the other hand, the validity of the OZI rule for $s\phi\phi$ couplings is generally not guaranteed in the L σ M. In Eq. (3.1), the two terms with the coefficients λ_1 and κ_d give generally OZI-forbidden $s\phi\phi$ -couplings. In the following, among their component couplings, we consider only the coupling for the octet pseudoscalar ϕ -meson (that is, $\langle\phi\rangle = 0$), since only the $\pi\pi$ and KK decay channels are relevant. These terms include the following:

$$-2\kappa_d\langle s'\phi^2\rangle + \kappa_d\langle s'\rangle\langle\phi^2\rangle - \lambda_1\langle\Sigma s'\rangle\langle\phi^2\rangle. \quad (3.3)$$

In Eq. (3.3), the second and third terms are OZI-forbidden. They are explicitly represented in the ideal bases (σ_n and σ_s), and Eq. (3.3) is rewritten into the form

$$-2\kappa_d\langle s'\phi^2\rangle - \{\sqrt{2}(\lambda_1 a - \kappa_d)\sigma_n + (\lambda_1 b - \kappa_d)\sigma_s\}\langle\phi^2\rangle. \quad (3.4)$$

On the other hand, in the ideal bases, the elements of the squared mass matrix of the iso-singlet scalar mesons are given by

$$\begin{aligned} m_{\sigma_n\sigma_n}^2 &= \mu^2 + \lambda_1(2a^2 + b^2) + 4\lambda_1 a^2 + 6\lambda_2 a^2 - 2\kappa_d b \\ m_{\sigma_s\sigma_s}^2 &= \mu^2 + \lambda_1(2a^2 + b^2) + 2\lambda_1 b^2 + 6\lambda_2 b^2 \\ m_{\sigma_n\sigma_s}^2 &= 2\sqrt{2}a(\lambda_1 b - \kappa_d). \end{aligned} \quad (3.5)$$

In the case of $\lambda_1 b = \kappa_d$, the value of $m_{\sigma_n\sigma_s}^2$ vanishes and the physical $\sigma(\sigma')$ becomes identical to the ideal state $\sigma_n(\sigma_s)$. Moreover, in this case for σ_s , as shown in Eq. (3.4), the OZI-forbidden second and third terms in Eq. (3.3) cancel each other, and the OZI-allowed first term predicts $g_{\sigma_s\pi\pi} = 0$, giving the vanishing decay width $\Gamma_{\sigma'\pi\pi} = 0$. In Fig. 1 this case corresponds to the zero-points on the f_K/f_π -axis of the $\Gamma_{\sigma'\pi\pi}$ -curves. Furthermore, in this case for σ_n , the second and third terms approximately cancel each other, and their sum becomes small comparatively to the first term, and the OZI-rule is almost satisfied.

The region of the values of f_K/f_π chosen in §3.1 is close to this case ($\lambda_1 b = \kappa_d$), and thus the OZI-forbidden term is expected to be small. For example, in the case that $f_\pi = 0.093$ GeV, $f_K/f_\pi = 1.394$ and $m_\sigma = 0.585$ GeV, the parameters a, b, λ_1 and κ_d are determined, respectively, as 0.0658 GeV, 0.1176 GeV, 13.03 and 1.518 GeV. The coefficients of the OZI-forbidden terms for σ_s and σ_n in Eq. (3.4) are given, respectively, by $\lambda_1 b - \kappa_d = 0.0135$ GeV and $\lambda_1 a - \kappa_d = -0.662$ GeV. These values are much smaller than the coefficients $2\kappa_d = 3.04$ GeV of the OZI-allowed first term in Eq. (3.4), which may be considered as a typical OZI-allowed coupling term in L σ M.

Thus we may conclude that the situation of the approximate validity of the OZI rule, which was the basis of our assignment of σ and $f_0(980)$ in §2, is actually realized in SU(3) L σ M with our classified members.

^{*)} Due to the T_3^3 -breaking pattern of f , s acquires the vacuum expectation value $s_0 = \Sigma = \text{diag}\{a, a, b\}$ of the T_3^3 -breaking pattern. Correspondingly, actual scalar and pseudoscalar mass spectra include, through the λ_1, λ_2 and κ_d -terms, higher order effects of the T_3^3 -breaking of the f matrix, and strictly GMO mass formula is expected to be valid only approximately in the L σ M.

§4. Concluding Remarks

In this paper we have investigated the possibility of classification of the new scalar nonet, $\sigma(600)$, $\kappa(900)$, $a_0(980)$ and $f_0(980)$. First by assuming the approximate validity of the OZI rule for the decay property of $f_0(980)$, the $\sigma(600)$ and $f_0(980)$ were supposed to be ideal mixing states, $n\bar{n}$ and $s\bar{s}$, respectively, of the scalar σ -nonet. It was then pointed out that the mass value of the iso-singlet flavor-octet state, obtained from the orthogonal transformation, satisfies the Gell-Mann Okubo mass formula. Furthermore, it was shown that the experimental masses and widths of members of this scalar σ -nonet are consistent with those of the scalar nonet predicted by (the tree level calculation of) the $L\sigma M$, as shown in Table I.

This result implies that the chiral symmetry plays a stronger role than ever thought in understanding the strong interaction, not only for deriving the low energy theorems through the non-linear realization, but also for explaining the spectroscopy and reactions related to all the mesons with masses below and around ~ 1 GeV through the linear realization.

We now give supplementary discussions related to the present problem. It is often argued that the validity of the $L\sigma M$ and the existence of the σ -meson *as a chiral partner of the π -meson* are not acceptable, since the phenomenological pattern of the ten low energy coefficients of the $O(p^4)$ -level of chiral perturbation theory (ChPT)^{31)–35)} is not reproduced by the $L\sigma M$. Surely, the framework of ChPT is useful for relating phenomenologically the various *low energy* phenomena concerning the Nambu-Goldstone π -meson octet mutually, with the ten parameters. However, the above argument seems too excessive and not appropriate, since it is based only on the results of analyses of indirect experiments with much lower energy than m_σ . Whether or not a resonance exists should be investigated directly by experiments with sufficiently high energy to produce the relevant resonance. As a matter of fact, $\sigma(600)$ can now be directly observed both in $\pi\pi$ scattering and in production processes.^{36)–40) 3), 41)–43)} Furthermore, the parameters in the $L\sigma M$ describe the physics in the *resonance energy region as well as the low energy region*, and the contribution of the σ -meson to low energy quantities can be predicted with no *new* free parameters.^{*)}

On the other hand it has been discussed⁴⁵⁾ that in the framework of ChPT the effect of the σ -meson can be taken into account through the $O(p^4)$ and $O(p^6)$ Lagrangian. However, such an approach seems clearly to have no predictive power regarding the properties of the scalar σ -meson nonet itself *as a chiral partner of the π -nonet*.

Finally we would like to mention that, as we have pointed out previously,^{1), 27)} the σ -nonet treated in this paper should be discriminated from the 3P_0 -nonet. The σ -nonet is assigned the quantum numbers $(L, S) = (0, 0)$ in the “relativistic LS -

^{*)} In this connection we refer the reader to an interesting work which points out that the width and form factor of K_{l4} -decay, as well as the $\pi\pi$ -phase shift δ_0^0 in low energy region, are also approximately described by $SU(3)L\sigma M$,⁴⁴⁾ as in the case of chiral perturbation theory.

coupling scheme.”

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